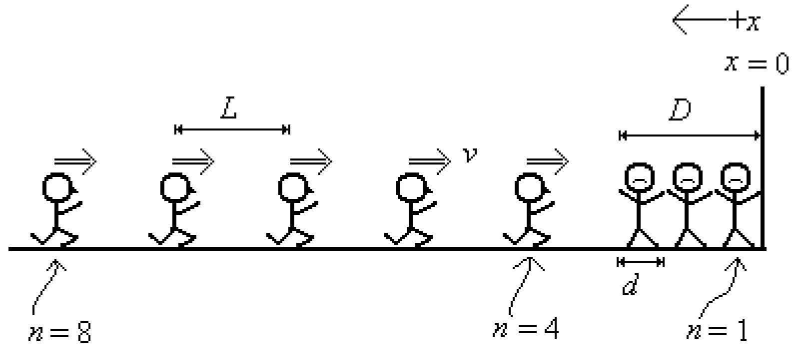
Advanced Placement Physics

Ch 02A : Solutions to Summer Assignment

created: 2008 0814; revised: 2008 0814

H&R 8e #02-010



Let n index the people (so that n = 1 is the first person, a distance d from the door at the start).
Let n_S be the number of people who have stopped (n_S = 3 in the picture).
Let D be the thickness of the layer of people already at the door. Clearly, $D = n_S d$.

At t = 0, the position of person n is simply nL. And from then, it decreases linearly. So it can be given by the expression

$$x_n(t) = \begin{cases} nL - vt & \text{for } t < t_n \\ nd & \text{for } t \geq t_n \end{cases} \qquad (1.1)$$

where t_n is the time the nth person reaches his/her stopping place. We use continuity to find $nL - vt_n = nd$ implying

$$t_n = n\frac{L-d}{v} \qquad (1.2)$$

Similarly, we can find out how many people have stopped:

$$n_S = \left\lfloor \frac{vt}{L-d} \right\rfloor \qquad (1.3)$$

where $\lfloor x \rfloor$ is the "floor function" (the largest integer less than or equal to x, so $\lfloor 2.3 \rfloor = 2$).

Now we can see how the thickness depends on time:

$$D(t) = \left\lfloor \frac{vt}{L-d} \right\rfloor d \qquad (1.4)$$

(a) By dealing with "average" values, we can essentially ignore the floor function. Then D is clearly proportional to t and grows at a rate

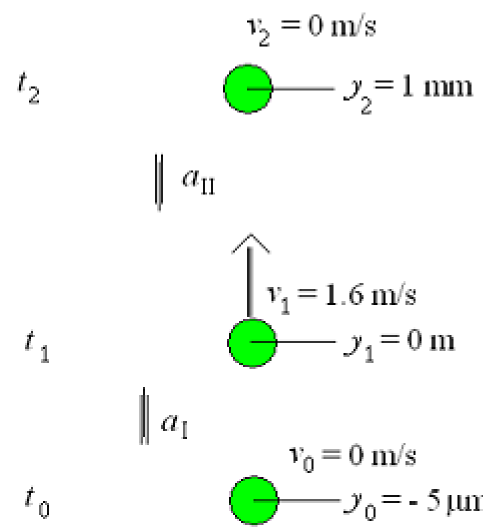
$$R = \frac{vd}{L-d} = \frac{3.5\,\frac{\text{m}}{\text{s}}\,0.25\text{ m}}{1.75\text{ m} - 0.25\text{ m}} = \boxed{0.583\,\frac{\text{m}}{\text{s}}} \qquad (1.5)$$

(b) If this is the average rate of growth, then $D = Rt$ and so

$$t_f = \frac{D_f}{R} = \frac{5\text{ m}}{0.583\,\frac{\text{m}}{\text{s}}} = \boxed{8.58\text{ s}}$$

Incidentally, there is a simulation of this situation on the class website at the same place the solutions are stored.

H&R 8e #02-28



(a) Launch:

$$v_1^2 = v_0^2 + 2a_I(y_1 - y_0)$$

So

$$a_I = \frac{v_1^2 - v_0^2}{2(y_1 - y_0)} = \frac{\left(1.6\,\frac{m}{s}\right)^2 - \left(0\,\frac{m}{s}\right)^2}{2\left(0\text{ m} - \left(-5\times10^{-6}\text{ m}\right)\right)}$$

$$a_I = 256\,000\,\frac{m}{s^2} \approx \boxed{26\,000\,g}$$

(b) Deceleration: (NB that a

$$v_2^2 = v_1^2 + 2(-a_{II})(y_2 - y_1)$$

So

$$a_{II} = -\frac{v_2^2 - v_1^2}{2(y_2 - y_1)} = -\frac{\left(0\,\frac{m}{s}\right)^2 - \left(1.6\,\frac{m}{s}\right)^2}{2\left(10^{-3}\text{ m} - 0\text{ m}\right)}$$

$$a_{II} = 1280\,\frac{m}{s^2} \approx \boxed{131\,g}$$

H&R 8e 02-042

(a) This is another one where the algebra might get grungy. Let's reason this out first. What does it mean to just avoid a collision? It means that their positions must overlap and the train must be moving equal to or slower than the locomotive. (If it's going faster, then the next instant, they'll collide anyway.)

x_T	position of train
v_T	velocity of train
$x_{T0} = 0$	initial position of train
$v_{T0} = 44.7$ m/s	initial velocity of train
a_T	acceleration of train
$x_T = x_{T0} + v_{T0}t + \frac{1}{2}(-a_T)t^2 = v_{T0}t - \frac{1}{2}a_T t^2$	
$v_T = v_{T0} + (-a_T)t$	

x_L	position of locomotive
v_L	velocity of locomotive
$x_{L0} = 676$ m	initial position of locomotive
$v_{L0} = 8$ m/s	initial velocity of locomotive
$a_L = 0$	acceleration of locomotive
$x_L = x_{L0} + v_{L0}t + \frac{1}{2}a_L t^2 = x_{L0} + v_{L0}t$	
$v_L = v_{L0}$	

Note that I've assumed $a_T > 0$, and put in the minus sign explicitly.

At the particular moment of collision, we have

$$t = t_C; \quad x_T = x_L; \quad v_T = v_L$$

So we can solve for t_C as

$$v_{L0} = v_{T0} - a_T t_C$$

$$t_C = \frac{v_{T0} - v_{L0}}{a_T}$$

$$v_{T0}\frac{(v_{T0} - v_{L0})}{a_T} - \frac{1}{2}a_T\left(\frac{(v_{T0} - v_{L0})}{a_T}\right)^2 = x_{L0} + v_{L0}\left(\frac{(v_{T0} - v_{L0})}{a_T}\right)$$

$$v_{T0}(v_{T0} - v_{L0}) - \frac{1}{2}(v_{T0} - v_{L0})^2 = x_{L0}a_T + v_{L0}(v_{T0} - v_{L0})$$

$$v_{T0}^2 - v_{T0}v_{L0} - \frac{1}{2}v_{T0}^2 - \frac{1}{2}v_{L0}^2 + v_{T0}v_{L0} = x_{L0}a_T + v_{T0}v_{L0} - v_{L0}^2$$

$$a_T = \frac{\frac{1}{2}v_{T0}^2 + \frac{1}{2}v_{L0}^2 - v_{T0}v_{L0}}{x_{L0}}$$

$$= \frac{(v_{T0} - v_{L0})^2}{2x_{L0}}$$

$$= \frac{(44.7\,\frac{\text{m}}{\text{s}} - 8\,\frac{\text{m}}{\text{s}})^2}{2 \times 676\,\text{m}}$$

$$= \boxed{0.996\,\frac{\text{m}}{\text{s}^2}}$$

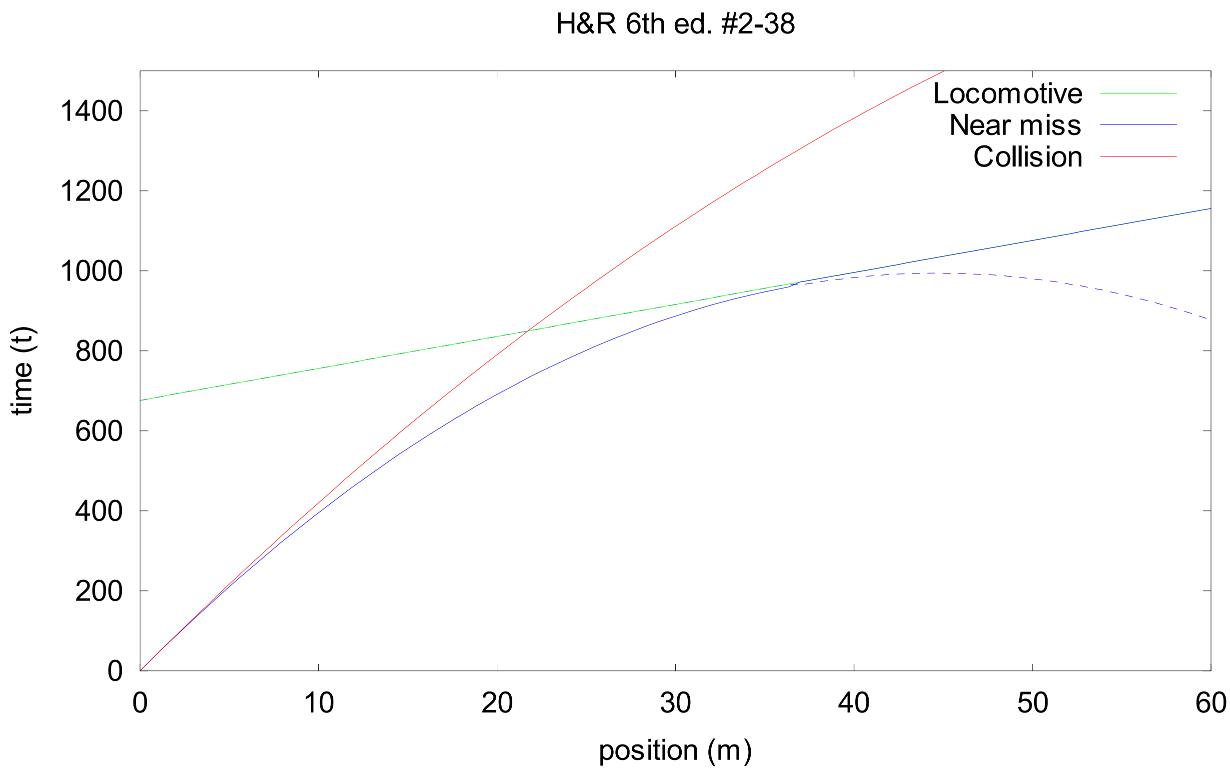
A simpler method:

Take it from the point of view of the locomotive. In this frame, the locomotive is at rest. We'll take $+x$ to be forward. The passenger train has an initial position of x_0 = -676 m, an initial velocity of v_0 = 36.7 m/s (the difference between their speeds as measured by an observer on the ground), and an acceleration of a. In this frame, the train must just come to rest at the moment it reaches $x = 0$.

$v^2 = v_0^2 + 2a\left(x - x_0\right)$ becomes $a = \frac{\cancel{v}^2 - v_0^2}{2\left(\cancel{x} - x_0\right)} = \frac{-v_0^2}{2x_0} = \boxed{-0.996\,\frac{\text{m}}{\text{s}^2}}$ and the time is simply

$$t = \frac{\cancel{v} - v_0}{a} = \boxed{36.84\,\text{s}}$$

(b)



H&R 8e #02-046

(a) Basic equation:

$$y = y_0 + (-v_0)(t - t_0) + \tfrac{1}{2}(-g)(t - t_0)^2$$

When it hits, $t = t_1$ and $y = y_1$, so

$$y_1 = y_0 + (-v_0)t_1 + \tfrac{1}{2}(-g)t_1^2$$

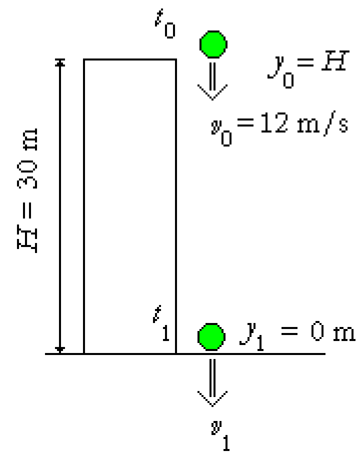
or

$$0 = H - v_0 t_1 - \tfrac{1}{2} g t_1^2$$

This can be solved graphically:

$$\mathtt{Y_1 = 30-12X-0.5*9.8*X^2}$$

which yields a zero at $t_1 = \boxed{1.54\text{ s}}$



(b) The other basic equation:

$$v_1^2 = v_0^2 + 2(-g)(y_1 - y_0)$$

which becomes

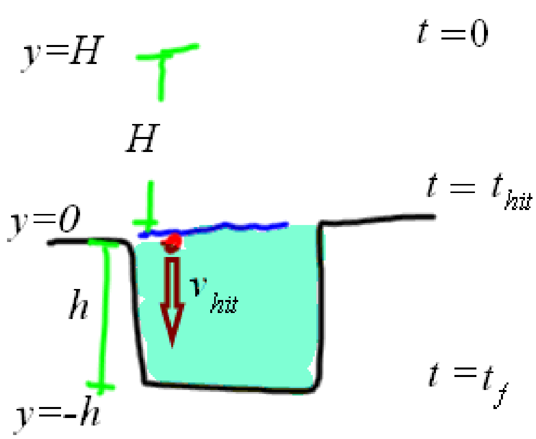
$$v_1^2 = v_0^2 + 2gH = \left(12\,\tfrac{\text{m}}{\text{s}}\right)^2 + 2\left(9.8\,\tfrac{\text{m}}{\text{s}^2}\right)30\text{ m} = 732\,\tfrac{\text{m}^2}{\text{s}^2}$$

Taking the root,

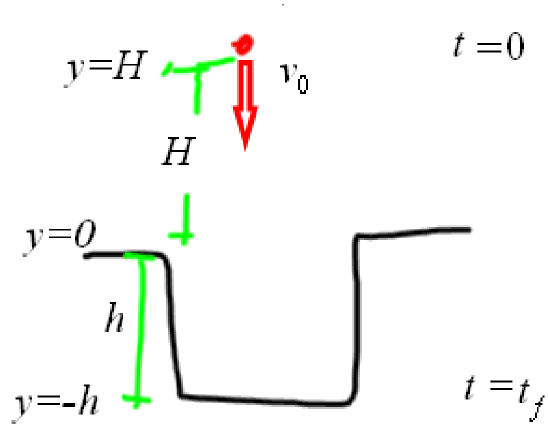
$$v_1 = 27.1\,\tfrac{\text{m}}{\text{s}}$$

and, of course, is directed downward as indicated on the drawing.

H&R 8e #02-074



filled pool (a)



emptied pool (b)

(a) We can find t_{hit} pretty easily since before hitting, the ball obeys

$$y = H - \tfrac{1}{2} g t^2$$

$$\Rightarrow 0 = H - \tfrac{1}{2} t_{hit}^2$$

$$\Rightarrow t_{hit} = \sqrt{\frac{2H}{g}}$$

Likewise, the velocity is relatively easy:

$$v = v_{start} - gt$$

$$\Rightarrow v_{hit} = -g t_{hit}$$

Since it now moves at constant speed (unphysical!), we have

$$v_{hit} = \frac{\Delta y}{\Delta t} = \frac{-h-0}{t_f - t_h}$$

$$\Rightarrow -\sqrt{2gH} = -\frac{h}{t_f - \sqrt{\frac{2H}{g}}}$$

$$\Rightarrow h = \sqrt{2gH}\left\{t_f - \sqrt{\frac{2H}{g}}\right\} = \sqrt{2\left(9.8\,\tfrac{\text{m}}{\text{s}^2}\right)5.20\text{ m}}\left\{4.8\text{ s} - \sqrt{\frac{2(5.2\text{ m})}{9.8\,\frac{\text{m}}{\text{s}^2}}}\right\}$$

$$\Rightarrow h = \boxed{38.1\text{ m}}$$

The average velocity can be found by using the definition:

$$v_{avg} = \frac{\Delta y}{\Delta t} = \frac{-h-H}{t_f - 0} = \frac{-38.1\text{ m} - 5.2\text{ m}}{4.80\text{ s}} = -9.03\,\tfrac{\text{m}}{\text{s}}$$

This clearly has magnitude (b) of 9.03 m/s and a direction (c) of downward.

Now it's almost trivial, because the position is simply

$$y = H - v_0 t - \tfrac{1}{2} g t^2$$
$$\Rightarrow -h = H - v_0 t_f - \tfrac{1}{2} g t_f^2$$
$$\Rightarrow v_0 = \frac{H + h - \tfrac{1}{2} g t_f^2}{t_f} = -v_{avg} - \tfrac{1}{2} g t_f = -\left(-9.03\,\tfrac{\text{m}}{\text{s}}\right) - \tfrac{1}{2}\left(9.8\,\tfrac{\text{m}}{\text{s}^2}\right) 4.80\text{ s}$$
$$\Rightarrow v_0 = \boxed{-14.5\,\tfrac{\text{m}}{\text{s}}}$$

Here, I assumed (apparently incorrectly) that the ball was thrown downward, but the presence of that final negative means the ball was thrown at (d) 38.6 m/s (e) upward.